AP® Physics C: Mechanics
2016 Free-Response Questions

CollegeBoard

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ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS	
Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, 1 eV $= 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ $\left(\text{N·m}^2\right)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol·K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s^2
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	1 u $= 1.66 \times 10^{-27}$ kg $= 931$ MeV$/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}$ J·s $= 4.14 \times 10^{-15}$ eV·s
	$hc = 1.99 \times 10^{-25}$ J·m $= 1.24 \times 10^3$ eV·nm
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}$ C$^2/\left(\text{N·m}^2\right)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^9$ $\left(\text{N·m}^2\right)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T·m)/A
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}$ (T·m)/A
1 atmosphere pressure,	1 atm $= 1.0 \times 10^5$ N$/\text{m}^2 = 1.0 \times 10^5$ Pa

UNIT SYMBOLS	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	$0°$	$30°$	$37°$	$45°$	$53°$	$60°$	$90°$
$\sin\theta$	0	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2	0
$\tan\theta$	0	$\sqrt{3}/3$	3/4	1	4/3	$\sqrt{3}$	∞

The following assumptions are used in this exam.
  I.   The frame of reference of any problem is inertial unless otherwise stated.
  II. The direction of current is the direction in which positive charges would drift.
  III. The electric potential is zero at an infinite distance from an isolated point charge.
  IV. All batteries and meters are ideal unless otherwise stated.
  V.   Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{J} = \int \vec{F} \, dt = \Delta \vec{p}$$

$$\vec{p} = m\vec{v}$$

$$\left| \vec{F}_f \right| \le \mu \left| \vec{F}_N \right|$$

$$\Delta E = W = \int \vec{F} \cdot d\vec{r}$$

$$K = \frac{1}{2} mv^2$$

$$P = \frac{dE}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

$$\Delta U_g = mg\Delta h$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$I = \int r^2 dm = \sum mr^2$$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$v = r\omega$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$K = \frac{1}{2} I\omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$$\vec{F}_s = -k\Delta\vec{x}$$

$$U_s = \frac{1}{2} k (\Delta x)^2$$

$$x = x_{max} \cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$T_p = 2\pi \sqrt{\frac{\ell}{g}}$$

$$\left| \vec{F}_G \right| = \frac{Gm_1 m_2}{r^2}$$

$$U_G = -\frac{Gm_1 m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$\left| \vec{F}_E \right| = \frac{1}{4\pi\varepsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$E_x = -\frac{dV}{dx}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

$$\Delta V = \frac{Q}{C}$$

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$$

$$R = \frac{\rho\ell}{A}$$

$$\vec{E} = \rho\vec{J}$$

$$I = Nev_d A$$

$$I = \frac{\Delta V}{R}$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = I\Delta V$$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$$\vec{F}_M = q\vec{v} \times \vec{B}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{\ell} \times \hat{r}}{r^2}$$

$$\vec{F} = \int I \, d\vec{\ell} \times \vec{B}$$

$$B_s = \mu_0 n I$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$U_L = \frac{1}{2} LI^2$$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \dfrac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell w h$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \dfrac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \dfrac{a}{c}$

$\cos\theta = \dfrac{b}{c}$

$\tan\theta = \dfrac{a}{b}$

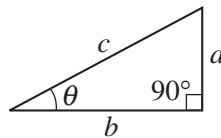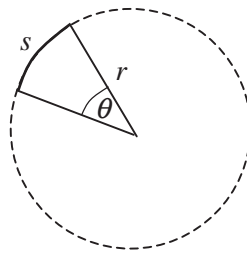$A =$ area
$C =$ circumference
$V =$ volume
$S =$ surface area
$b =$ base
$h =$ height
$\ell =$ length
$w =$ width
$r =$ radius
$s =$ arc length
$\theta =$ angle

CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}\left[\sin(ax)\right] = a\cos(ax)$$

$$\frac{d}{dx}\left[\cos(ax)\right] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$\vec{A}\bullet\vec{B} = AB\cos\theta$

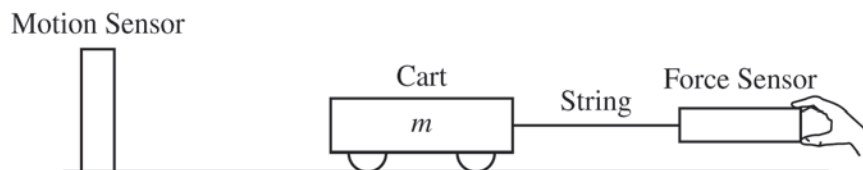$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

Mech.1.

A cart of mass *m* is pulled along a level dynamics track as shown above. A force sensor is attached to the cart with a string and used to measure the horizontal force exerted on the cart to the right. A motion sensor is used to measure the acceleration of the cart with the positive direction toward the right. Friction is not negligible.

(a) On the dot below, which represents the cart, draw and label the forces (not components) that act on the cart. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

●

A student pulls the force sensor with a constant force, and the cart accelerates. This is repeated for several trials, with a different constant force used for each trial. The data are recorded in the table below.

Trial	1	2	3	4	5
Force sensor reading (N)	0.32	0.38	0.44	0.50	0.60
Acceleration $\left(\text{m/s}^2\right)$	0.12	0.22	0.33	0.50	0.70

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(b)

    i. On the grid below, plot data points for the acceleration of the cart as a function of the force sensor reading. Clearly scale all axes. Draw a straight line that best represents the data.

Force Sensor Reading (N)
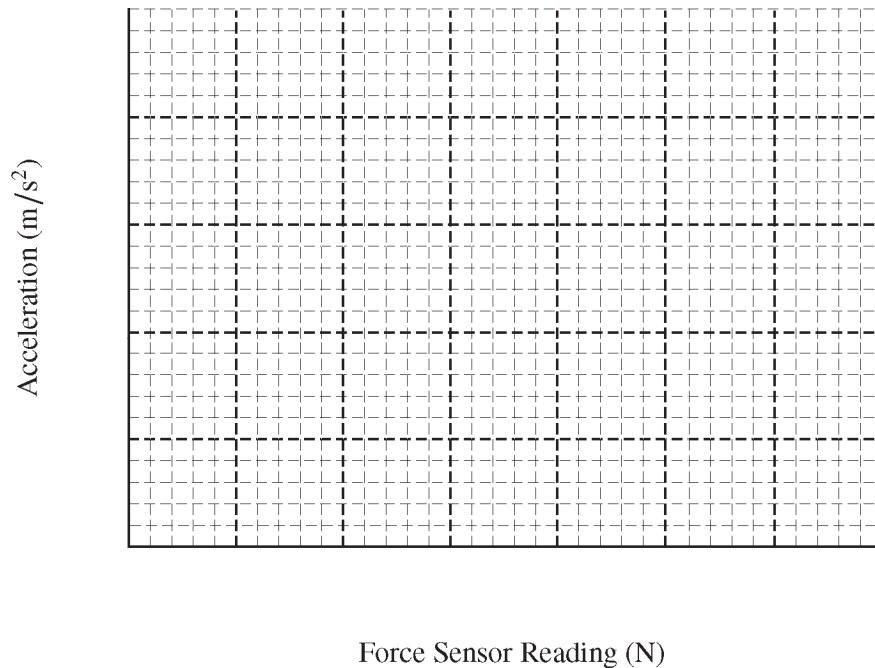
    ii. Using the straight line from the graph, calculate the mass of the cart.

    iii. Using the straight line from the graph, determine the magnitude of the force of friction.

The above experiment is repeated by using a constant force sensor reading of 0.45 N. The cart starts from rest at time $t = 0$ s and is pulled for a time of 2.0 s along the dynamics track.

(c)

    i. Determine the acceleration of the cart.

    ii. The string breaks at time $t = 2.0$ s. Calculate the time it takes for the cart to stop after the string breaks.

The experiment and analysis in parts (a) and (b) are repeated with a cart that has the same mass but a greater force of friction.

(d)

    i. Will the slope of your new line be greater than, less than, or equal to the slope of your line in part (b)i?

        ____ Greater than   ____ Less than   ____ Equal to

    ii. Will the horizontal intercept of your new line be greater than, less than, or equal to the horizontal intercept of your line in part (b)i?

        ____ Greater than   ____ Less than   ____ Equal to

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Mech.2.

A block of mass 2*M* rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where *B* is a positive constant and *x* is the displacement from equilibrium for the spring. A block of mass 3*M* and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass 2*M* and 3*M*, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass 2*M*   Block of Mass 3*M*

●   ●

The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance *D*, as shown above. Express your answers to parts (b), (c), and (d) in terms of *M*, *B*, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

(d) Derive an expression for the maximum distance *D* that the spring is compressed.

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(e)

    i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

        ____ Left   ____ Right   ____ The net force on the block of mass $2M$ is zero.

        Justify your answer.

    ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

        ____ The magnitude of the net force is greater on the block of mass $2M$.

        ____ The magnitude of the net force is greater on the block of mass $3M$.

        ____ The magnitude of the net force on each block has the same nonzero value.

        ____ The magnitude of the net force on each block is zero.

        Justify your answer.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

    ____ Yes   ____ No

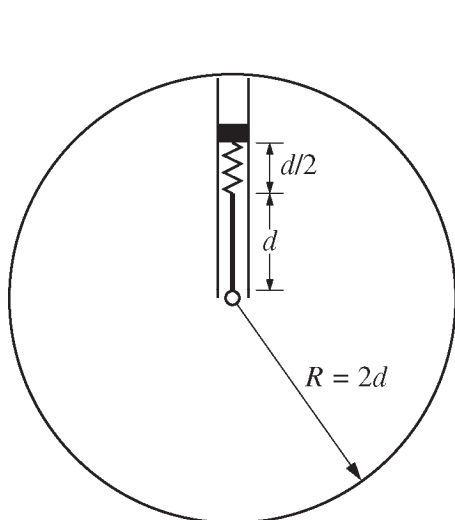Justify your answer.

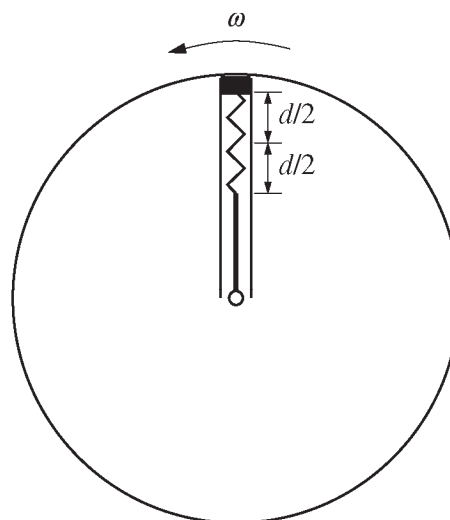Figure 1   Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\dfrac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\dfrac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

(b)

    i. Determine an expression for the rotational inertia of the block around the axis of the platform.

    ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.
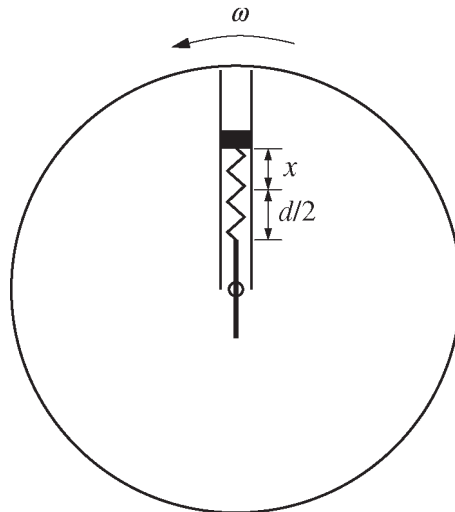
Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing   _____ Decreasing   _____ Staying the same

Justify your answer.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive   _____ Negative   _____ No work

Justify your answer.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.
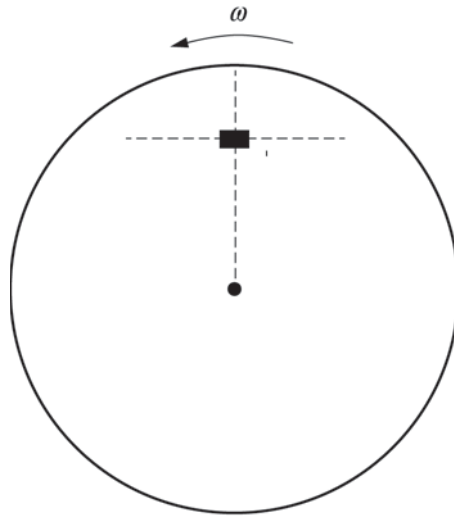

Figure 4

STOP

END OF EXAM